Futures Markets and the Theory of the Firm Under Price Uncertainty
Author(s): Gershon Feder, Richard E. Just and Andrew Schmitz
Source: *The Quarterly Journal of Economics*, Vol. 94, No. 2 (Mar., 1980), pp. 317-328
Published by: Oxford University Press
Stable URL: http://www.jstor.org/stable/1884543
Accessed: 02/06/2013 12:18

FUTURES MARKETS AND THE THEORY OF THE FIRM UNDER PRICE UNCERTAINTY*

GERSHON FEDER
RICHARD E. JUST
ANDREW SCHMITZ

This paper examines the behavior of a competitive firm under price uncertainty where a futures market exists for the commodity produced by the firm. Working with the Sandmo approach, we found that production decisions depend only on the futures market price and input costs; the subjective distribution of future spot price affects only the firm's involvement in futures trading. Conditions are then determined under which a firm will either hedge, speculate by buying futures contracts, or speculate by selling futures contracts. The results indicate that an important social benefit derived from the existence of a futures market is to eliminate output fluctuations due to variation in producers' subjective distributions of future spot price.

The theory of the firm under price uncertainty has been the subject of a considerable number of studies. While the papers by Sandmo [1971], Leland [1972], Batra and Ullah [1974], and others have undoubtedly advanced the understanding of the decision making of firms under uncertainty, the theory is not complete unless forward delivery contracts are considered. Such contracts, referred to as "futures," are playing an increasingly important role in a number of commodity markets.

While a substantial body of literature exists on the subject of futures markets and their implications, surprisingly few attempts have been made to incorporate futures trading in a generalized theory of production under price uncertainty. McKinnon [1967] presents a model where both price and output are random, but planned production is not a decision variable. This is in sharp contrast to the standard theory of the firm where the production decision plays a central role. In addition, the decision-makers in McKinnon's model are assumed to operate within a mean-variance framework. The works of Rutledge [1972] and Ward and Fletcher [1971] present improvements over McKinnon's model by incorporating several time periods, storage, trade in both input and output, etc. But production is not considered explicitly. Attitudes toward risk are reflected, as in McKinnon's model, via the mean-variance tradeoff. The mean-variance approach is subject to fairly restrictive assumptions, and the more general expected utility approach would seem to be preferable.

The purpose of the present paper is to incorporate the possibility

* Giannini Foundation Paper No. 518. This paper benefited from the suggestions of the Associate Editor and an anonymous referee.

The Quarterly Journal of Economics, March 1980 0033-5533/80/0094-0317$01.00

of buying and selling futures into the model of the competitive firm under price uncertainty developed by Sandmo [1971] and Batra and Ullah [1974]. Both the production decision and the decision regarding the extent of trade in futures contracts are treated explicitly. The discussion focuses on the individual decision-maker; thus, the mechanism of price determination is taken to be exogenous.[1] Specifically, the paper addresses the question of how production and futures trade are related to prevailing futures contract prices, the subjective distribution of the future cash price, and production costs. It is demonstrated how the existence of a futures market leads to results that are quite different from those predicted by the Sandmo-Batra-Ullah theory of the firm under uncertainty. In particular, it is demonstrated that the production decision is separated from the futures trading decision and is not subject to risk considerations.

I. THE MODEL

Suppose that, at the time of decision making, a producer determines his production by choosing optimal input levels. At the same time as input purchases are made, he also decides whether to sell contracts for future delivery (i.e., delivery at the date production is realized) in an amount that will provide either a complete or partial hedge against declining product prices. Also, the producer can speculate by selling more contracts than his planned production level or by buying futures contracts. The futures contract price is known at the time of decision making, but uncertainty exists with respect to the spot price that will prevail at contract termination.

To make the analysis manageable, the output of the firm is assumed nonrandom; i.e., output is determined by the optimal input levels through a standard production function. This is also the assumption made by Sandmo, and Batra and Ullah, who dealt with price uncertainty and the theory of the firm. Thus, the producer knows the amount of output that will be available to meet future contract deliveries.[2] This assumption realistically depicts the output of metals and forest products for which active futures markets exist. Although this assumption may be questionable for nonirrigated crops where

1. For a discussion on price determination in commodity markets and futures markets, see Peck [1976].

2. As mentioned above, Ward and Fletcher [1971] and Rutledge [1972] more specifically assumed a linear production technology. McKinnon [1967] alternatively considered output to be random but assumed that input decisions are exogenous to the model. Regarding price uncertainty, however, the McKinnon model deals with only a very special case of the model developed in this paper, namely, the case where expected, discounted price is equal to current price.

yields fluctuate due to such factors as weather variability, it may approximate the situation for irrigated crops where output is much more certain at the time of planting. Also, for cattle feedlot operations (where futures trading is used extensively), output is known with a relatively high degree of certainty, since losses due to death are usually small.

For notational purposes, let

K = input level
$F(K)$ = production function, with the properties $F' > 0$, $F'' < 0$
X = volume of future contracts sold (or purchased if negative)
C = unit cost of inputs
P_0 = futures contract price
r = interest rate
P = product price at termination date (or future spot price)
U = utility function, with the properties $U' > 0$, $U'' < 0$.

The profit is given by $P[F(K) - X] + (P_0 \cdot X - C \cdot K)(1 + r)$ and is the value of output not committed by futures contracts, plus the cash receipts (or outlays) from the sale of futures contracts, minus the cost of inputs. The last two terms are corrected for interest costs (or revenues), since these transactions take place at present, while profits are evaluated at the end of the period. The objective of the producer is to maximize his expected utility of profits at the end of the period,

$$(1) \qquad \max_{K,X} EU\{P[F(K) - X] + (P_0 \cdot X - C \cdot K)(1 + r)\},$$

where the decision variables are input level K and the volume of futures trade X. The first-order conditions for expected utility maximization are

$$(2) \qquad \frac{\partial EU}{\partial K} = E\{U'[PF' - (1 + r)C]\} = 0$$

$$(3) \qquad \frac{\partial EU}{\partial X} = E\{U'[P_0(1 + r) - P]\} = 0.$$

For purposes of later derivation, these conditions can be written compactly as $E(U'A) = E(U'B) = 0$, where $A \equiv PF' - (1 + r)C$ and $B \equiv P_0(1 + r) - P$. Following Feder [1977, Lemma 4], one can show that

$$(4) \qquad A = -F'B;$$

whence the Hessian matrix can be written as

$$H = \begin{bmatrix} E(U''A^2) + F''E(U'P) & -E(U''A^2)/F' \\ -E(U''A^2)/F' & E(U''A^2)/(F')^2 \end{bmatrix},$$

and second-order conditions for a maximum hold, since

$$\Delta \equiv |H| = F''E(U'P)E(U''A^2)/(F')^2 > 0.$$

II. The Influence of a Futures Market on Producer Uncertainty and Production

In excellent papers on the theory of the firm under price uncertainty, Sandmo and Batra and Ullah demonstrated that an increase in price uncertainty (represented by a mean-preserving spread of the P distribution) causes a decline in production,[3] while an increase in expected price $\overline{P}$ causes an increase in planned production. However, in their model only production decisions are included. In the following section the factors affecting output levels are examined when the firm can react to the presence of risk not only by output changes but also through changes in its involvement in futures trade.

PROPOSITION 1. With the existence of a futures market, the production decision is not affected by changes in the subjective distribution of the future spot price, nor is it influenced by the decision-maker's degree of risk aversion. Output is positively affected by the prevailing futures price and responds negatively to increases in input cost.

Proof. From (4), it can be determined that

$$F' = C/P_0. \tag{5}$$

Hence, planned production is completely determined by production costs C and the current price of futures contracts P_0; the distribution of P plays no role; neither does the utility function (i.e., the degree of risk aversion). Differentiation of (5) yields $dK/dC = 1/(F''P_0) < 0$ and $dK/dP_0 = -C/(F''P_0^2) > 0$.

Q.E.D.

REMARK 1. Proposition 1 establishes an important property of the model that is essential for the derivation of the remaining results. That is, with the presence of a futures market, a complete separation is maintained between the production decision and the futures trading decision. Production is decided as if the prevailing

3. Actually, Sandmo [1971] concluded that the impact of increased uncertainty on output is ambiguous, but Ishii [1977] demonstrated, using Sandmo's own model, that output is reduced when risk increases.

futures price is the only relevant price. Since there is no uncertainty about that price, production is not affected by attitudes toward risk or changes in expectations regarding the unknown future spot price. This is in contrast to the normal model of the firm under price uncertainty[4] where an increase in uncertainty causes a decline in production, while an increase in expected price $\bar{P}$ induces an increase in planned production. The reason for the difference in results stems from the fact that, in earlier models, the only way in which a firm could cope with changes in price uncertainty was by altering the volume of output. In the present model the firm can enter the futures market, while optimal production is unaltered. *The existence of a functioning futures market thus serves to eliminate output fluctuations* (*i.e.*, *output changes along a given supply curve*) *that are due to changes in producer's subjective distributions of the future spot price.* It should be noted, however, that the separation property indicated above would not hold if production were subject to uncertainty (in addition to price uncertainty). Thus, the results apply to those commodities for which production is essentially nonrandom.

REMARK 2. The results of Proposition 1 shed some light on the age-old question, repeated by Peck [1976], "Do producers use futures prices (or the local market equivalent) in their production decision?" According to the result in (5), the futures price is the driving force affecting producers' production decisions. Even though the current spot price may play a large role in determining producers' subjective expectations about future spot prices, planned production is totally unaffected by future price expectations; only involvement in the futures market is possibly altered.[5]

III. THE ROLE OF SUBJECTIVE INFORMATION, PRICES, AND COSTS IN FUTURES MARKET TRADING

Having concluded in the previous section that uncertainty and risk aversion do not enter in the production decision, it follows necessarily that these factors affect the volume and direction of futures

4. See, for example, Sandmo [1971], Ishii [1977], and Batra and Ullah [1974].

5. The result of Proposition 1 justifies McKinnon's assumption that the production decision is exogenous to his model; production is truly unaffected by the unknown future spot price. McKinnon [1967, p. 846], however, adopts this assumption as a simplification and asserts that "... farmers would normally make their planting decision for a particular crop dependent on expected future prices." This assertion is refuted by Proposition 1.

trading. Once output is given, the trading decision becomes a portfolio selection problem where the risky asset is the amount of product available to the decision-maker at contract delivery (and output maturity) date, while the nonrisky commodity is the cash to be earned by sale of futures contracts at the present futures price. A question of central importance is to determine when is it advantageous for a producer to hedge some or all of his production against a fall in price. Similarly, it is interesting to see whether or not (and under what circumstances) producers may be induced to enter the market as speculators. For the purposes of this paper, a partial hedge is defined as the sale of futures contracts in an amount that is positive but less than planned production; a hedge commits all output in the futures market, and speculation implies the sale of futures contracts in an amount larger than planned output or buying futures contracts in any amount. From the model specified, one finds:

PROPOSITION 2. If discounted (subjective) expectations of the future spot price are less than the present futures contract price, then a producer will optimally behave as a speculator selling futures contracts in excess of planned production. If discounted expectations are equal to the present futures price, the producer will place a hedge. If discounted expected price exceeds the current futures price, a producer will either place a partial hedge, not enter the futures market at all, or speculate by buying futures contracts.

Proof. Adding and subtracting $\overline{P}$ within the square brackets of equation (3) implies that

$$[P_0(1+r) - \overline{P}]EU' = E[U'(P - \overline{P})] \equiv \sigma, \tag{6}$$

where σ is defined by $\sigma \equiv \text{cov}(U',P)$. Define the volume of product that is subject to uncertain price as $Z \equiv F(K) - X$. It can be shown, following Feder [1977, Lemma 1], that $Z\,\sigma < 0$ for $Z \neq 0$. (One can easily show that $Z = 0$ if and only if $\sigma = 0$.) However, from (6), it is obvious that the sign of σ is the same as the sign of $P_0(1+r) - \overline{P}$; hence,

$$Z \lesseqgtr 0 \quad \text{as} \quad P_0 \gtreqless \overline{P}/(1+r). \tag{7}$$

Q.E.D.

In the following we investigate the effects of changes in the distribution of P and the futures price P_0. In order to investigate the impact of changes in degree of uncertainty, we introduce a parameter γ, which measures the level of risk. An increase in γ implies a mean preserving spread of the distribution of P.[6] Assuming that absolute risk aversion is nonincreasing[7] and that the maximand in (1) is concave in γ, one finds the following:

PROPOSITION 3. A small increase in the degree of uncertainty relating to the future spot price implies, ceteris paribus, that (1) the volume of futures contracts sold declines ($dX/d\gamma < 0$) if the current futures price is greater than the discounted, expected future spot price; (2) futures trading is unaffected if the current futures price is equal to discounted future spot price expectations; and (3) the volume of futures contracts sold increases or the volume purchased declines if the current futures contract price is less than the discounted, expected future spot price.

Proof. Following Feder [1977, Theorem 1], it is known that an increase in the degree of uncertainty will change the level of $Z = [F(K) - X]$ in a direction opposite to the sign of Z. Combining this with Proposition 2 implies that

$$\frac{dZ}{d\gamma} \lesseqqgtr 0 \quad \text{as} \quad P_0 \lesseqqgtr \frac{\overline{P}}{1+r}.$$

Using the definition of Z, one obtains $dZ/d\gamma = (dF/d\gamma) - (dX/d\gamma)$. But by Proposition 1, $dF/d\gamma = 0$, and thus

$$\frac{dX}{d\gamma} \gtreqqless 0 \quad \text{as} \quad P_0 \lesseqqgtr \frac{\overline{P}}{1+r}.$$

Q.E.D.

REMARK 3. The reader may note that Proposition 3 does not hold in full generality since second-order conditions need not be satisfied with respect to γ. However, the results in Proposition 3 reflect a plausible pattern of behavior for a risk-averse decision-maker. The absolute volume of Z represents the extent of involvement in an uncertain (risky) activity, and as uncertainty increases, Z approaches zero either from above [if $P_0(1 + r) < \overline{P}$] or from below [if $P_0(1 + r) > \overline{P}$]. Hence, as uncertainty increases, the extent of involvement in the uncertain activity decreases.[8]

6. See Rothschild and Stiglitz [1970], Sandmo [1971], and Feder [1977].

7. This property has been advocated by Arrow [1965]; it was subsequently adopted by Sandmo [1971] and Batra and Ullah [1974].

8. Note that the results of Proposition 3 are consistent with McKinnon's results. Assuming that $P_0(1 + r) = \overline{P}$, he finds that optimal forward sales are given by mean output plus a term that depends on the correlation between output and price and on the variances of output and price. Thus, case 2 of Propositions 2 and 3, where all output is committed in futures contracts, corresponds to the special case of McKinnon's results where output and price are uncorrelated.

PROPOSITION 4. Ceteris paribus, a change in the volume of futures contracts sold (purchased) is inversely (directly) related to future spot price expectations ($\overline{P}$).

Proof. If one recalls Proposition 1, $dF(K)/(d\overline{P}) = 0$, and thus $dX/d\overline{P} = -dZ/d\overline{P}$, where Z has been defined earlier. Following Feder [1977, Theorem 2], one can see that a change in $\overline{P}$ will cause a change in the optimal level of Z in the same direction. This implies that X changes in a direction opposite to $\overline{P}$, i.e., $dX/d\overline{P} < 0$.

Q.E.D.

PROPOSITION 5. With a ceteris paribus increase in the current futures contract price (P_0), a speculator [$X < 0$ or $X > F(K)$] will increase the volume of contracts sold or decrease the volume purchased. A hedger [$X = F(K)$] will also increase the volume of contracts sold and thus become a speculator. A partial hedger [$0 < X < F(K)$] may increase, decrease, or not change his futures market involvement; but if his absolute risk aversion is constant, then his futures contract sales will increase.

Proof. Differentiation of (2) and (3) yields

$$\frac{dX}{dP_0} = -\frac{1}{\Delta}\Bigg[(1+r)E(U')\,E\,U''A^2) + (1+r)E(U')F''E(U'P) - \frac{\delta}{F'}F''E(U'P)\Bigg], \tag{8}$$

where $\delta \equiv (1+r)\,XE(U''A)$. The first two terms in square brackets are negative given the concavity of F and U. Following Feder [1977, Lemma 2], one can also show that $ZE(U''A) \geq 0$ where equality implies either the fact that $Z = 0$ or constant absolute risk aversion. Thus,

$$\delta \begin{cases} < 0 & \text{if } X < 0 \text{ or } Z = F(K) - X < 0 \\ > 0 & \text{if } X > 0 \text{ and } Z > 0, \text{ i.e., } 0 < X < F(K) \\ = 0 & \text{if } X = 0,\ Z = 0, \text{ or absolute risk aversion is constant.} \end{cases}$$

Hence, one can conclude from (8) that $dX/dP_0 > 0$ if $X \leq 0$ or $Z \leq 0$, but $dX/dP_0 \leq 0$ is a possibility when $0 < X < F(K)$ unless absolute risk aversion is constant.

Q.E.D.

REMARK 4. The results reported in Proposition 5 can be better understood when equation (8) is written as

$$\frac{dK}{dP_0} = \frac{F'dK}{dP_0} - \frac{(1-r)}{\Delta} E(U')F''E(U'P) + \frac{\delta F''}{\Delta F'} E(U' P).$$

Note that the first term on the right-hand side is, in fact, the increase in output due to the increase in P_0, which has already been shown to be positive. It follows that, in the cases where δ is zero or negative, the amount of futures contracts sold is increased (or the amount purchased is decreased) by more than the increase in production. One may refer to the first term on the right-hand side as the "production effect," while the other terms are the "substitution effect." The latter effect describes the volume of goods that originally would have been subject to the uncertain future spot price but that are transferred to the riskless transaction of futures sales as the latter activity becomes more rewarding (in terms of expected utility) at the margin. For partial hedgers, however, the substitution effect may not be positive, since the higher level of profits that are related to a higher P_0, combined with decreasing absolute risk aversion, implies that the firm is less reluctant to engage in risky activities. Accordingly, the firm may, at the margin, switch away from futures contract sales. If this impact is strong enough, not only will new production be reserved for sale at the uncertain price, but also some futures contracts will be bought (or old contracts recalled), which implies that the overall volume of futures sold by the firm becomes lower than in a situation of lower P_0. This latter possibility will not be observed when firms have an approximately constant level of absolute risk aversion, since in that case no "reverse substitution" takes place.

PROPOSITION 6. As production costs increase, futures contract sales are decreased for hedgers and partial hedgers, while speculators buying futures contracts will increase purchases.

Proof. Differentiating (2) and (3) obtains

$$\frac{dX}{dC} = -\frac{1}{\Delta}\left[\frac{F''E(UP)}{F'}\zeta - \frac{E(U''A^2)}{F'}(1+r)E(U')\right], \tag{9}$$

where $\zeta = (1+r)\,KE(U''A)$. As in the proof of Proposition 5 relating to $E(U''A)$, one finds that $\zeta \leq 0$ if $Z \geq 0$; if $\zeta \leq 0$, then $dX/dC < 0$, and the results of Proposition 6 follow from Proposition 1.

Q.E.D.

REMARK 5. According to Proposition 1, as costs are increased, the operation of the firm is reduced so that reliance on a futures market to reduce profit variability is needed to a lesser extent. For a speculator selling futures contracts, however, the expected difference in the current futures and expected future spot prices may still be sufficient so that speculation is not reduced. Indeed, production costs may be sufficiently high so that production ceases but speculation in selling futures contracts would continue in the case where $\overline{P} > P_0\,(1 + r)$ provided that $F'(0)$ is finite. Similarly, $dX/dC > 0$ implies that speculation in buying futures contracts could increase, while higher costs cause production to cease.

IV. The Case Where Price Expectations Are Influenced by Current Prices

Propositions 3 through 5 carry the implicit assumptions that a change in the current futures contract price does not influence the subjective distribution of future spot prices or, vice versa, that changes in subjective future price distributions do not affect current futures prices. It seems reasonable, however, as evidenced by the popularity of adaptive expectations models, that price expectations may change as current prices change. To consider this possibility, suppose that future spot price expectations are influenced by the current futures contract price according to some function ψ, $\overline{P} = \psi\,[(1 + r)P_0]$, where $0 < \psi' \leq 1$. The latter assumption implies that any movement in current price is not reversed or amplified in the discounted future price expectation.

PROPOSITION 7. When movements in current futures contract prices induce positively related movements in future spot price expectations, optimal planned production responds to current futures prices just as in the case where price expectations are not influenced by current futures prices. If, in addition, the induced movements in discounted price expectations are no greater than the movements in current futures prices, then the volume of futures contracts sold will increase (or the volume purchased will increase) as the current futures contract price increases (or, equivalently, as future spot price expectations increase) for all speculators selling futures contracts, hedgers, and decision-makers with constant absolute risk aversion.

Proof. The first part of Proposition 7 follows trivially from equation (5). As for the second part, differentiation of (2) and (3) obtains

$$(10)\qquad \frac{dX}{dP_0} = -\frac{1}{\Delta}\left\{(1+r)E(U')E(U''A^2) + F''E(U'P)\left[(1+r)E(U') - \frac{\mu}{F'}\right]\right\},$$

where

$$(11)\quad \mu \equiv (1+r)F'\psi'E(U') + [F(K)\psi' + X(1-\psi')](1+r)E(U''A).$$

Using (11), however, one finds

$$\left[(1+r)E(U') - \frac{\mu}{F'}\right] = (1+r)E(U')\left[1 - \psi' - \frac{F(K)\psi' + X(1-\psi')}{F'}\frac{E(U''A)}{E(U')}\right] \geq 0$$

if $\psi \leq 1$ and $E(U''A) \leq 0$. But, again, using $ZE(U''A) \geq 0$, as in the proof of Proposition 5, implies that $E(U''A) \leq 0$ when $Z = F(K) - X \leq 0$ and, following Feder, $E(U''A) = 0$ regardless of Z when absolute risk aversion is constant. Therefore, substituting back into (10) obtains $dX/dP_0 > 0$.

Q.E.D.

V. Conclusions

In this paper the theory of the firm under price uncertainty has been extended by including futures market trading as an activity of the firm in addition to the physical production of the good. The firm's involvement in both production and futures trading in response to price uncertainty has been determined jointly. Once the possibility is allowed for futures market trading, the production decision becomes independent of the distribution of the uncertain price; this is contrary to the predictions of standard theory. Production is shown to depend on the prevailing futures price. These results have an important implication regarding policies that seek to affect production, since as demonstrated in this paper, attempts to influence subjective distributions will result in changes in the extent of futures trade; but production will remain as originally planned if futures prices are unaffected.

The volume of futures trade is shown to be inversely related to the degree of uncertainty that is the expected result when the futures market serves as an insurance device. It is demonstrated, however, that risk-averse producers may be induced to speculate by either buying futures contracts or selling more futures contracts than their planned production. Coupling the results of Propositions 3 through 6 also explains the existence of both pure speculators (nonproducers) and pure producers (nonfutures-market traders). High production costs (e.g., little knowledge of how to produce) and definite subjective information (less dispersion of the future spot price distribution or more separation between futures price and future spot price expectations) tend toward pure speculation; lower production costs and less subjective market information tend toward pure production.

WORLD BANK
UNIVERSITY OF CALIFORNIA, BERKELEY, AND BRIGHAM YOUNG UNIVERSITY
UNIVERSITY OF CALIFORNIA, BERKELEY

REFERENCES

Arrow, K. J., "Aspects of the Theory of Risk Bearing," Yrjo Johnson Lectures, Helsinki, 1965.

Batra, R. N., and A. Ullah, "Competitive Firm and the Theory of Input Demand Under Price Uncertainty," *Journal of Political Economy*, LXXXII (May/June 1974), 537–48.

Feder, G., "The Impact of Uncertainty in a Class of Objective Functions," *Journal of Economic Theory*, XVI (Dec. 1977), 504–12.

Ishii, Y., "On the Theory of the Competitive Firm Under Price Uncertainty: Note," *American Economic Review*, LXVII (Sept. 1977), 768–69.

Leland, H., "Theory of the Firm Facing Random Demand," *American Economic Review*, LXII (June 1972), 278–91.

McKinnon, R. I., "Futures Markets, Buffer Stocks, and Income Stability for Primary Producers," *Journal of Political Economy*, LXXV (Dec. 1967), 844–61.

Peck, A. E., "Futures Markets, Supply Response, and Price Stability," this *Journal*, XC (Aug. 1976), 407–23.

Rothschild, M., and J. E. Stiglitz, "Increasing Risk: I. A Definition," *Journal of Economic Theory*, II (Sept. 1970), 225–43.

Rutledge, D. J., "Hedgers' Demand for Futures Contracts: A Theoretical Framework with Applications to the United States Soybean Complex," *Stanford Food Research Institute Studies in Agricultural Economics, Trade, and Development*, XI (1972), 237–56.

Sandmo, A., "On the Theory of the Competitive Firm Under Price Uncertainty," *American Economic Review*, LXI (March 1971), 65–73.

Ward, R. N., and L. B. Fletcher, "A Micro Model of Optimal Futures and Cash Market Positions," *American Journal of Agricultural Economics*, LIII (Feb. 1971), 71–78.